Accelerated Geom/ Adv Alg Name____________ ID: 1

Matrices Review NO CALCULATOR Date____________ Period____

Simplify. Write "undefined" for expressions that are undefined.

1) $\begin{bmatrix} 2 & -1 & 2 & 0 \end{bmatrix} - \left(\begin{bmatrix} 1 & 1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} 0 & -4 & 0 & 4 \end{bmatrix}\right)$

$\begin{bmatrix} 2 & -1 & 2 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 5 & -1 & -2 \end{bmatrix}$

$\begin{bmatrix} 1 & -6 & 3 & 2 \end{bmatrix}$

2) $3\begin{bmatrix} 1 & 0 \\ 1 & -2 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} 2 & 2 \\ -3 & 1 \\ -1 & 5 \end{bmatrix}$

$\begin{bmatrix} 3 & 0 \\ 3 & -6 \\ 0 & -3 \end{bmatrix} - \begin{bmatrix} 2 & 2 \\ -3 & 1 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 6 & -7 \\ 1 & -8 \end{bmatrix}$

3) $\begin{bmatrix} 1 & -2 & 3 \\ 1 & 6 & -5 \end{bmatrix} - \left(-3\begin{bmatrix} 2 & 5 & 3 \\ 1 & 3 & -6 \end{bmatrix}\right)$

$\begin{bmatrix} 1 & -2 & 3 \\ 1 & 6 & -5 \end{bmatrix} - \begin{bmatrix} -6 & -15 & -9 \\ -3 & -9 & 18 \end{bmatrix}$

$\begin{bmatrix} 7 & 13 & 12 \\ 4 & 15 & -23 \end{bmatrix}$

4) $\begin{bmatrix} 5 & -6 & 4 \end{bmatrix} - 2\begin{bmatrix} 4 & 5 & -3 \end{bmatrix}$

$\begin{bmatrix} 5 & -6 & 4 \end{bmatrix} + \begin{bmatrix} -8 & -10 & +6 \end{bmatrix}$

$\begin{bmatrix} -3 & -16 & 10 \end{bmatrix}$

5) $\left(\begin{bmatrix} 6 & 3 \\ 5 & -6 \end{bmatrix} - \begin{bmatrix} 6 & -5 \\ 2 & -5 \end{bmatrix}\right) \cdot \begin{bmatrix} -6 & 5 & -5 \\ 4 & 2 & -6 \end{bmatrix}$

$\begin{bmatrix} 0 & 8 \\ 3 & -1 \end{bmatrix}$ 2×2 $\begin{bmatrix} -6 & 5 & -5 \\ 4 & 2 & -6 \end{bmatrix}$ 2×3

2×3

$\begin{bmatrix} \underline{32} & \underline{16} & \underline{-48} \\ \underline{-18-4} & \underline{15-2} & \underline{-15+6} \end{bmatrix}$

$\begin{bmatrix} 32 & 16 & -48 \\ -22 & 13 & -9 \end{bmatrix}$

6) $2 \cdot \left(\begin{bmatrix} -4 \\ -2 \end{bmatrix} \cdot \begin{bmatrix} 6 & -5 & -5 \end{bmatrix}\right)$

2×1 1×3

$2\left(\begin{bmatrix} -24 & 20 & 20 \\ -12 & 10 & 10 \end{bmatrix}\right)$

$\begin{bmatrix} -48 & 40 & 40 \\ -24 & 20 & 20 \end{bmatrix}$

7) $-3\left(\begin{bmatrix} -3 & -4 \\ 0 & 3 \\ -1 & 0 \end{bmatrix} - \begin{bmatrix} 6 \\ 3 \end{bmatrix}\right)$

3(undefined

8) $\begin{bmatrix} -3 & -3 \\ 2 & -3 \\ -1 & -6 \end{bmatrix} + \begin{bmatrix} -5 & 1 \\ -5 & 5 \\ -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} -6 & -6 \\ 6 & 1 \end{bmatrix}$

3x2 2x2 3x2

$\begin{bmatrix} 30+6 & 30+1 \\ 30+30 & 30+5 \\ 6-6 & 6-1 \end{bmatrix}$

$\begin{bmatrix} -3 & -3 \\ 2 & -3 \\ -1 & -6 \end{bmatrix} + \begin{bmatrix} 36 & 31 \\ 60 & 35 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 33 & 28 \\ 62 & 32 \\ -1 & -1 \end{bmatrix}$

9) $\begin{bmatrix} -6 & 0 \\ -1 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -3 & -6 & -3 \\ 3 & 5 & -5 \end{bmatrix}$

$\begin{bmatrix} -6 & 6 \\ -1 & 7 \end{bmatrix} \cdot \begin{bmatrix} -3 & -6 & -3 \\ 3 & 5 & -5 \end{bmatrix}$

2x2 2x3

2x3

$\begin{bmatrix} 18+18 & 36+30 & 18+-30 \\ 3+21 & 6+35 & 3+-35 \end{bmatrix}$

$\begin{bmatrix} 36 & 66 & -12 \\ 24 & 41 & -32 \end{bmatrix}$

10) $\begin{bmatrix} -6 & -4 \\ 4 & 6 \\ 5 & -6 \end{bmatrix} - \left(\begin{bmatrix} -1 & -3 \\ -4 & -5 \\ -4 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -5 \\ -6 & 1 \\ 4 & -6 \end{bmatrix} \right)$

$\begin{bmatrix} -6 & -4 \\ 4 & 6 \\ 5 & -6 \end{bmatrix} - \begin{bmatrix} -1 & -8 \\ -10 & -4 \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} -5 & 4 \\ 14 & 10 \\ 5 & -2 \end{bmatrix}$

Evaluate each determinant.

11) $\begin{vmatrix} -3 & 1 \\ 4 & 4 \end{vmatrix}$

$-12-4$

-16

12) $\begin{vmatrix} -5 & -2 \\ -5 & -5 \end{vmatrix}$

$25-10$

15

13) $\begin{vmatrix} -2 & -2 & -3 \\ 2 & -3 & 0 \\ -3 & -3 & -2 \end{vmatrix} \begin{matrix} -2 & -2 \\ 2 & -3 \\ -3 & -3 \end{matrix}$

$(-12+0+18)-(-27+0+8)$

$6-(-19)$

25

14) $\begin{vmatrix} 1 & -2 & -2 \\ 1 & -5 & -1 \\ 5 & -4 & 0 \end{vmatrix} \begin{matrix} 1 & -2 \\ 1 & -5 \\ 5 & -4 \end{matrix}$

$(0+10+8)-(50+4+0)$

$18-(54)$

-36

$\begin{array}{r} 54 \\ -18 \\ \hline 36 \end{array}$

15) $\begin{vmatrix} 0 & 4 & 4 \\ -5 & 0 & -4 \\ -3 & 3 & -3 \end{vmatrix} \begin{matrix} 0 & 4 \\ -5 & 0 \\ -3 & 3 \end{matrix}$

$(0+48+60)-(0+0+60)$

$108-60$

48

16) $\begin{vmatrix} 2 & -1 & 5 \\ -5 & -5 & -1 \\ 2 & -2 & 3 \end{vmatrix} \begin{matrix} 2 & -1 \\ -5 & -5 \\ 2 & -2 \end{matrix}$

$(-30+2+50)-(-50+4+15)$

$22-(-31)$

53

52
-30

456
-19
31
22

Use Cramer's Rule to solve each system.

17) $4x-3y=1$
$-x+4y=3$

det $=16-3$

$\begin{bmatrix} 4 & -3 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$x=\dfrac{\begin{vmatrix} 1 & -3 \\ 3 & 4 \end{vmatrix}}{13}$ $\quad Y=\dfrac{\begin{vmatrix} 4 & 1 \\ -1 & 3 \end{vmatrix}}{13}$

$X=\dfrac{4-(-9)}{13}$ $\quad Y=\dfrac{12-(-1)}{13}$

$X=\dfrac{13}{13}$ $\quad Y=\dfrac{13}{13}$

$X=1$ $\quad Y=1$

$(1,1)$

18) $-5x+3y=4$
$6x+2y=-6$

$\begin{bmatrix} -5 & 3 \\ 6 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 4 \\ -6 \end{bmatrix}$

$\det A=-10-18$
$=-28$

$X=\dfrac{\begin{vmatrix} 4 & 3 \\ -6 & 2 \end{vmatrix}}{-28}$ $\quad Y=\dfrac{\begin{vmatrix} -5 & 4 \\ 6 & -6 \end{vmatrix}}{-28}$

$\left(-\dfrac{13}{14}, -\dfrac{3}{14}\right)$

$X=\dfrac{8-(-18)}{-28}$ $\quad Y=\dfrac{30-24}{-28}$

$X=\dfrac{26}{-28}$

$X=-\dfrac{13}{14}$ $\quad Y=\dfrac{6}{-28}$

19) $-3x-2y=8$
$x+6y=-14$

$\begin{bmatrix} -3 & -2 \\ 1 & 6 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix}=\begin{bmatrix} 8 \\ -14 \end{bmatrix}$

14
×3
2

$\det A=-18+(+2)$
$\det A=-16$

$Y=\dfrac{\begin{vmatrix} -3 & 8 \\ 1 & -14 \end{vmatrix}}{-16}$

$X=\dfrac{\begin{vmatrix} 8 & -2 \\ -14 & 6 \end{vmatrix}}{-16}$ $\quad Y=\dfrac{42-8}{-16}$

$X=\dfrac{48-28}{-16}$ $\quad Y=\dfrac{34}{-16}$

$Y=-\dfrac{17}{8}$

$X=\dfrac{20}{-16}$

$X=-\dfrac{5}{4}$

$\left(-\dfrac{5}{4}, -\dfrac{17}{8}\right)$

20) $x+4y=11$
$-2x-6y=-6$

$\begin{bmatrix} 1 & 4 \\ -2 & -6 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix}=\begin{bmatrix} 11 \\ -6 \end{bmatrix}$

$\det A=-6-(-8)$
$\det A=2$

$X=\dfrac{\begin{vmatrix} 11 & 4 \\ -6 & -6 \end{vmatrix}}{2}$ $\quad Y=\dfrac{\begin{vmatrix} 1 & 11 \\ -2 & -6 \end{vmatrix}}{2}$

$X=\dfrac{-66-(-24)}{2}$ $\quad Y=\dfrac{-6-(-22)}{2}$

$X=\dfrac{-42}{2}$ $\quad Y=\dfrac{16}{2}$

$X=-21$ $\quad Y=8$

$(-21, 8)$

$\begin{array}{r}184\\ \times 3\\ \hline 252\end{array}$

21) $4x + 2y - 3z = -3$
$-x - 4y = -16$
$-3x + 2y + z = 21$

$$\begin{bmatrix}4 & 2 & -3\\ -1 & -4 & 0\\ -3 & 2 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}-3\\ -16\\ 21\end{bmatrix}$$

$$\left|\begin{array}{ccc|cc}4 & 2 & -3 & 4 & 2\\ -1 & -4 & 0 & -1 & -4\\ -3 & 2 & 1 & -3 & 2\end{array}\right|$$

$(-16+0+6)-(-36+0+-2) \rightarrow (-10)-(-38)$

determinant $= 28$

$$X = \left|\begin{array}{ccc|cc}-3 & 2 & -3 & -3 & 2\\ -16 & -4 & 0 & -16 & -4\\ 21 & 2 & 1 & 21 & 2\end{array}\right|$$

$(12+0+96)-(252+0+-32)$

$108-220$

$X = -\frac{112}{28} \rightarrow X = -4$

$\begin{array}{r}516\\ \times 9\\ \hline 144\end{array}$

$$Y = \left|\begin{array}{ccc|cc}4 & -3 & -3 & 4 & -3\\ -1 & -16 & 0 & -1 & -16\\ -3 & 21 & 1 & -3 & 21\end{array}\right|$$

$\begin{array}{r}196\\ 6\\ \hline 102\end{array}$

$(-64+0+63)-(-144+0+3)$

$-1-(-141)$

140

$\begin{array}{r}428\\ \times 5\\ \hline 140\end{array}$

$\begin{array}{r}21\\ \times 16\\ \hline 126\\ 21\\ \hline 336\end{array}$

$Y = \frac{140}{28} \rightarrow Y = 5$

$\begin{array}{r}32\\ \times 3\\ \hline 96\end{array}$ $\begin{array}{r}336\\ -102\\ \hline 234\end{array}$

$$Z = \left|\begin{array}{ccc|cc}4 & 2 & -3 & 4 & 2\\ -1 & -4 & -16 & -1 & -4\\ -3 & 2 & 21 & -3 & 2\end{array}\right|$$

$\begin{array}{r}32\\ \times 4\\ \hline 128\end{array}$

$(-336+96+6)-(-36-128-42)$

$-234-(-206)$

$\begin{array}{r}128\\ 42\\ 36\\ \hline 206\end{array}$ $\begin{array}{r}438\\ -206\\ \hline 232\end{array}$

$Z = -\frac{28}{28}$

$\begin{array}{r}234\\ -206\\ \hline 28\end{array}$

$Z = -1$

$(-4, 5, -1)$

22) $-3x + y + 5z = 13$
$-4x + 4z = 12$
$-3x + 2y + 3z = 5$

$$\begin{bmatrix}-3 & 1 & 5\\ -4 & 0 & 4\\ -3 & 2 & 3\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}13\\ 12\\ 5\end{bmatrix}$$

$$\left|\begin{array}{ccc|cc}-3 & 1 & 5 & -3 & 1\\ -4 & 0 & 4 & -4 & 0\\ -3 & 2 & 3 & -3 & 2\end{array}\right|$$

$(0+-12+-40)-(0+-24-12) \rightarrow -52-(-36)$

-16

$\begin{array}{r}452\\ -36\\ \hline 16\end{array}$

$\begin{array}{r}24\\ \times 5\\ \hline 120\end{array}$ $\begin{array}{r}23\\ \times 8\\ \hline 104\end{array}$

$$X = \left|\begin{array}{ccc|cc}13 & 1 & 5 & 13 & 1\\ 12 & 0 & 4 & 12 & 0\\ 5 & 2 & 3 & 5 & 2\end{array}\right|$$

$(0+20+120)-(0+104+36)$

$140-140 \rightarrow \frac{0}{-16}$

$X = 0$

$$Y = \left|\begin{array}{ccc|cc}-3 & 13 & 5 & -3 & 13\\ -4 & 12 & 4 & -4 & 12\\ -3 & 5 & 3 & -3 & 5\end{array}\right|$$

$\begin{array}{r}136\\ \times 3\\ \hline 108\end{array}$ $\begin{array}{r}12\\ \times 13\\ \hline 36\\ 12\\ \hline 156\end{array}$ $\begin{array}{r}156\\ 108\\ 100\\ \hline 364\end{array}$

$(-108+-156-100)-(-180-60-156)$

$-364-(-396)$

$\begin{array}{r}12\\ 15\\ \hline 60\\ 12\\ \hline 180\end{array}$

$Y = \frac{32}{-16} \rightarrow Y = -2$

$$Z = \left|\begin{array}{ccc|cc}-3 & 1 & 13 & -3 & 1\\ -4 & 0 & 12 & -4 & 0\\ -3 & 2 & 5 & -3 & 2\end{array}\right|$$

$\begin{array}{r}213\\ \times 8\\ \hline 104\end{array}$

$(0-36-104)-(0-72-20)$

$-140-(-92)$

$\begin{array}{r}1\cancel{40}\\ -92\\ \hline 48\end{array}$

$Z = -\frac{48}{-16}$

$Z = 3$

$(0, -2, 3)$

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23) $2y - 3z = -5$
$-2x - 2y + 4z = 14$
$2x - 3y + z = 1$

$$\begin{bmatrix} 0 & 2 & -3 \\ -2 & -2 & 4 \\ 2 & -3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ 14 \\ 1 \end{bmatrix}$$

$$\left|\begin{array}{ccc|cc} 0 & 2 & -3 & 0 & 2 \\ -2 & -2 & 4 & -2 & -2 \\ 2 & -3 & 1 & 2 & -3 \end{array}\right|$$

$(0+16-18)-(12+0-4)$ $=2-8$

determinant
-10

$$X = \frac{\left|\begin{array}{ccc|cc} -5 & 2 & -3 & -5 & 2 \\ 14 & -2 & 4 & 14 & -2 \\ 1 & -3 & 1 & 1 & -3 \end{array}\right|}{-10}$$

$14 \times 9 = 126$ $\quad 66 + 28 = 94$ $\quad 144 - 94 = 50$

$$x = \frac{(10+8+126)-(6+60+28)}{-10}$$

$$\frac{144-94}{-10}$$

$$x = \frac{50}{-10}$$

$x = -5$

$$Y = \frac{\left|\begin{array}{ccc|cc} 0 & -5 & -3 & 0 & -5 \\ -2 & 14 & 4 & -2 & 14 \\ 2 & 1 & 1 & 2 & 1 \end{array}\right|}{-10}$$

$14 \times 6 = 84$

$$y = \frac{(0-40+6)-(-84+0+10)}{-10}$$

$$\frac{-34-(-74)}{-10}$$

$$y = \frac{40}{-10}$$

$y = -4$

$$Z = \frac{\left|\begin{array}{ccc|cc} 0 & 2 & -5 & 0 & 2 \\ -2 & -2 & 14 & -2 & -2 \\ 2 & -3 & 1 & 2 & -3 \end{array}\right|}{-10}$$

$14 \times 4 = 56$

$$z = \frac{(0+56-30)-(20+0-4)}{-10}$$

$$\frac{26-16}{-10}$$

$$z = \frac{10}{-10}$$

$z = -1$

$(-5, -4, -1)$

24) $y - 4z = 6$
$-x - y - 4z = -1$
$4x - 2y - 5z = 13$

$$\begin{bmatrix} 0 & 1 & -4 \\ -1 & -1 & -4 \\ 4 & -2 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \\ 13 \end{bmatrix}$$

$$\left|\begin{array}{ccc|cc} 0 & 1 & -4 & 0 & 1 \\ -1 & -1 & -4 & -1 & -1 \\ 4 & -2 & -5 & 4 & -2 \end{array}\right|$$

$(0+-16+-8)-(16+0+5)$
$-24-21$

determinant
-45

$13 \times 4 = 52$

$$x = \frac{\left|\begin{array}{ccc|cc} 6 & 1 & -4 & 6 & 1 \\ -1 & -1 & -4 & -1 & -1 \\ 13 & -2 & -5 & 13 & -2 \end{array}\right|}{-45}$$

$$x = \frac{(30+-52+-8)-(+52+48+5)}{-45}$$

$$\frac{-30-(105)}{-45}$$

$$x = \frac{-135}{-45}$$

$x = 3$

$$Y = \frac{\left|\begin{array}{ccc|cc} 0 & 6 & -4 & 0 & 6 \\ -1 & -1 & -4 & -1 & -1 \\ 4 & 13 & -5 & 4 & 13 \end{array}\right|}{-45}$$

$16 \times 6 = 96$

$$Y = \frac{(0-96+52)-(16+0+30)}{-45}$$

$$Y = \frac{-44-46}{-45}$$

$$y = \frac{-90}{-45}$$

$y = 2$

$$Z = \frac{\left|\begin{array}{ccc|cc} 0 & 1 & 6 & 0 & 1 \\ -1 & -1 & -1 & -1 & -1 \\ 4 & -2 & 13 & 4 & -2 \end{array}\right|}{-45}$$

$$z = \frac{(0+-4+12)-(-24+0-13)}{-45}$$

$$z = \frac{8-(-37)}{-45}$$

$$z = \frac{45}{-45}$$

$z = -1$

$(3, 2, -1)$

Find the inverse of each matrix.

25) $\begin{bmatrix} -1 & 5 \\ 1 & -6 \end{bmatrix}$ $\frac{1}{6-5}\begin{bmatrix} -6 & -5 \\ -1 & -1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -6 & -5 \\ -1 & -1 \end{bmatrix}$

26) $\begin{bmatrix} 5 & 9 \\ 3 & 7 \end{bmatrix}$

$\begin{array}{r} 35 \\ -27 \end{array}$

$A^{-1} = \frac{1}{35-27}\begin{bmatrix} 7 & -9 \\ -3 & 5 \end{bmatrix}$

$\frac{1}{8}\begin{bmatrix} 7 & -9 \\ -3 & 5 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{7}{8} & -\frac{9}{8} \\ -3/8 & 5/8 \end{bmatrix}$

27) $\begin{bmatrix} 2 & -6 \\ 2 & -6 \end{bmatrix}$

$A^{-1} = \frac{1}{-12+(-12)}\begin{bmatrix} -6 & 6 \\ -2 & 2 \end{bmatrix}$

$\frac{1}{0}\begin{bmatrix} -6 & 6 \\ -2 & 2 \end{bmatrix}$

Inverse does not exist

28) $\begin{bmatrix} 0 & -5 \\ -1 & 5 \end{bmatrix}$

$A^{-1} = \frac{1}{0-5}\begin{bmatrix} 5 & 5 \\ 1 & 0 \end{bmatrix}$

$= \frac{1}{-5}\begin{bmatrix} 5 & 5 \\ 1 & 0 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -1 & -1 \\ -1/5 & 0 \end{bmatrix}$

Solve each equation. $X = A^{-1}B$

29) $\overset{A}{\begin{bmatrix} 3 & 3 \\ -6 & -4 \end{bmatrix}} B = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$A^{-1} = \frac{1}{-12+(+18)}\begin{bmatrix} -4 & -3 \\ 6 & 3 \end{bmatrix}$

$= \frac{1}{6}\begin{bmatrix} -4 & -3 \\ 6 & 3 \end{bmatrix}$

$= \begin{bmatrix} -4/6 & -3/6 \\ 6/6 & 3/6 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -2/3 & -1/2 \\ 1 & 1/2 \end{bmatrix}$

$B = \underset{2\times 2}{\begin{bmatrix} \boxed{-2/3 \quad -1/2} \\ 1 \quad 1/2 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} 3 \\ 4 \end{bmatrix}}$

$= \begin{bmatrix} -2+-2 \\ 3+2 \end{bmatrix}$

$B = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$

30) $\overset{A}{\begin{bmatrix} 10 & -5 \\ -9 & -6 \end{bmatrix}} = \begin{bmatrix} 0 & -5 \\ -1 & 4 \end{bmatrix} X$

$A^{-1} = \frac{1}{0-5}\begin{bmatrix} 4 & 5 \\ 1 & 0 \end{bmatrix}$

$= \frac{1}{-5}\begin{bmatrix} 4 & 5 \\ 1 & 0 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -4/5 & -1 \\ -1/5 & 0 \end{bmatrix}$

$\begin{bmatrix} -4/5 & -1 \\ -1/5 & 0 \end{bmatrix}\begin{bmatrix} 10 & -5 \\ -9 & -6 \end{bmatrix} = X$

$\begin{bmatrix} -8+9 & 4+6 \\ -2+0 & 1+0 \end{bmatrix} = X$

$\begin{bmatrix} 1 & 10 \\ -2 & 1 \end{bmatrix} = X$

31) $\begin{bmatrix} -8 & -8 \\ -10 & -11 \end{bmatrix} = \overset{A}{\begin{bmatrix} 2 & -6 \\ 3 & -10 \end{bmatrix}} Z$

$A^{-1} = \frac{1}{-20-(-18)} \begin{bmatrix} -10 & 6 \\ -3 & 2 \end{bmatrix}$

$= \frac{1}{-2} \begin{bmatrix} -10 & 6 \\ -3 & 2 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 5 & -3 \\ 3/2 & -1 \end{bmatrix}$

$\begin{bmatrix} 5 & -3 \\ 3/2 & -1 \end{bmatrix} \begin{bmatrix} -8 & -8 \\ -10 & -11 \end{bmatrix} = Z$

$\begin{bmatrix} -40+30 & -40+33 \\ -12+10 & -24/2+11 \end{bmatrix} = Z$

$\begin{bmatrix} -10 & -7 \\ -2 & -1 \end{bmatrix} = Z$

32) $\begin{bmatrix} 3 & -1 \\ -8 & 2 \end{bmatrix} Z = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$

$A^{-1} = \frac{1}{6-8} \begin{bmatrix} 2 & 1 \\ 8 & 3 \end{bmatrix}$

$= \frac{1}{-2} \begin{bmatrix} 2 & 1 \\ 8 & 3 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -1 & -1/2 \\ -4 & -3/2 \end{bmatrix}$

$Z = \underset{2\times 2}{\begin{bmatrix} -1 & -1/2 \\ -4 & -3/2 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} -2 \\ -2 \end{bmatrix}} = \underset{2\times 1}{\begin{bmatrix} 2+1 \\ 8+3 \end{bmatrix}}$

$\begin{bmatrix} 3 \\ 11 \end{bmatrix}$

33) Prove that the Matrices are inverses of each other.

$\overset{A}{\begin{bmatrix} -4 & 3 \\ -2 & 1 \end{bmatrix}} \overset{A^{-1}}{\begin{bmatrix} \frac{1}{2} & -\frac{3}{2} \\ 1 & -2 \end{bmatrix}}$

$A \cdot A^{-1} = I$

$A^{-1} A = I$

$A \cdot A^{-1}$

$\begin{bmatrix} -2+3 & 6+-6 \\ -1+1 & 3+-2 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^{-1} \cdot A$

$\begin{bmatrix} \frac{1}{2} & -3/2 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} -4 & 3 \\ -2 & 1 \end{bmatrix}$

$\begin{bmatrix} -2+3 & 3/2+-3/2 \\ -4+4 & 3+-2 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Since A·A⁻¹ and A⁻¹·A both equal the identity matrix, the matrices are inverses of each other.

34) Prove that the Matrices are inverses of each other.

$\overset{A}{\begin{bmatrix} 5 & 6 \\ -9 & -9 \end{bmatrix}} \overset{A^{-1}}{\begin{bmatrix} -1 & -\frac{2}{3} \\ 1 & \frac{7}{9} \end{bmatrix}}$

$A \cdot A^{-1}$

$\begin{bmatrix} -5+6 & -10/3 + 42/9 \\ +9+-9 & 6-7 \end{bmatrix}$

$-\frac{10}{3} \cdot \frac{3}{3}$

$\frac{-30}{9} + \frac{42}{9}$

$\frac{12}{9} = \frac{4}{3}$

$\begin{bmatrix} 1 & 4/3 \\ 0 & -1 \end{bmatrix}$

Since A·A⁻¹ does no equal the identity matrix, the matrices are not inverses of each other.

35) Prove that the Matrices are inverses of each other.

$$A = \begin{bmatrix} -8 & -1 & \frac{11}{2} \\ -17 & -2 & 12 \\ 1 & 0 & -\frac{1}{2} \end{bmatrix} \quad A^{-1} = \begin{bmatrix} -2 & 1 & 2 \\ -7 & 3 & -5 \\ -4 & -2 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A \cdot A^{-1}$

$$\begin{bmatrix} 16+7+-22 \;(+1) & -8-3-11 \;(-22) & \\ & & \\ & & \end{bmatrix}$$

23+-22

-22

-11-11

this element must equal 0.

Since $A \cdot A^{-1}$ does not equal the identity matrix the matrices are not inverses of each other.

36) Prove that the Matrices are inverses of each other.

$$A = \begin{bmatrix} -1 & 0 & 5 \\ -3 & -5 & -1 \\ 0 & 1 & 3 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 14 & -5 & -25 \\ -9 & 3 & 16 \\ 3 & -1 & -5 \end{bmatrix}$$

14 × 3 = 42

16 × 5 = 80

$A \cdot A^{-1}$

$$\begin{bmatrix} -14+0+15 & 5+0+-5 & 25+0-25 \\ -42+45-3 & 15+-15+1 & 75-80+5 \\ 0-9+9 & 0+3-3 & 0+16-15 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

14 × 5 = 70

$A^{-1} \cdot A$

$$\begin{bmatrix} 14 & -5 & -25 \\ -9 & 3 & 16 \\ 3 & -1 & -5 \end{bmatrix} \begin{bmatrix} -1 & 0 & 5 \\ -3 & -5 & -1 \\ 0 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} -14+15+0 & 0+25-25 & 70+5-75 \\ 9-9+0 & 0-15+16 & -45-3+48 \\ -3+3+0 & 0+5-5 & 15+1-15 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since $A^{-1} \cdot A$ and $A \cdot A^{-1}$ both equal the identity matrix, the matrices are inverses of each other.