

Name \_\_\_\_\_

Date \_\_\_\_\_ Period \_\_\_\_\_

Divide using synthetic division:

1)  $(2n^3 - 6n^2 - 26n - 61) \div (n - 6)$

$$\begin{array}{r|rrrr} 6 & 2 & -6 & -26 & -61 \\ & & 12 & 36 & 60 \\ \hline & 2 & 0 & 10 & -1 \end{array}$$

$2n^2 + 6n + 10 - \frac{1}{n-6}$

Expand:

3)  $(x-5)^3$

$$\begin{array}{cccc} 1 & 3 & 3 & 1 \\ 1 & 4 & 6 & 3 & 1 \\ 1 & 3 & 3 & 1 \\ 1 & 3 & 3 & 1 \\ 1 & 3 & 3 & 1 \end{array}$$

$x^3 - 15x^2 + 75x - 125$

2)  $(4x^5 - 40x^4 - 8x^2 + 86x - 58) \div (x - 10)$

$$\begin{array}{r|rrrrrr} 10 & 4 & -40 & 0 & -8 & 86 & -58 \\ & & 40 & 0 & 0 & -80 & 60 \\ \hline & 4 & 0 & 0 & -8 & 6 & 2 \end{array}$$

$4x^4 - 8x + 6 + \frac{2}{x-10}$

4)  $(3x + 2y)^4$

$$\begin{array}{cccc} 1 & 3 & 3 & 3 & 1 \\ 1 & 4 & 6 & 4 & 1 \\ 1 & 3 & 3 & 3 & 1 \\ 1 & 3 & 3 & 3 & 1 \\ 1 & 3 & 3 & 3 & 1 \end{array}$$

$81x^4 + 216x^3y + 216x^2y^2 + 96xy^3 + 16y^4$

Factor completely: SOAPP

5)  $x^4 - 2x^3 + 27x - 54$

$$x^3(x-2) + 27(x-2)$$

$$(x^3 + 27)(x-2)$$

$(x-2)(x+3)(x^2 - 3x + 9)$

6)  $-15x^4 + 7x^2 + 2$

$$-1(15x^4 - 7x^2 - 2)$$

$$-1(15x^4 - 10x^2 + 3x^2 - 2)$$

$$-(5x^2(3x^2 - 2) + 1(3x^2 - 2))$$

$-(5x^2 + 1)(3x^2 - 2)$

Write a polynomial function with the following roots

7)  $-2i, 0$  (multiplicity of 3)

$$x^2 = (-2i)^2 \rightarrow x^2 = -4$$

$$x^2 = 4i^2 \rightarrow x^2 = -4$$

$$x^2 + 4 = 0$$

$f(x) = x^3(x^2 + 4)$

$f(x) = x^5 + 4x^3$

8)  $-3, 2 + \sqrt{5}$

$x = -3$

$x + 3 = 0$

$x = 2 + \sqrt{5}$

$x - 2 = \sqrt{5}$

$(x-2)^2 = (\sqrt{5})^2$

$f(x) = x^3 - x^2 - 13x - 3$

$$x^2 - 4x + 4 = 5$$

$$x^2 - 4x - 1 = 0$$

$x^2$	$-4x$	$-1$
$x^3$	$-4x^2$	$-x$
$3x^2$	$-12x$	$-3$

Simplify the following:

$$9) (6x - 5x^4 + 2x^2) - (7x^2 - 8x^4)$$

$$6x - 5x^4 + 2x^2 - 7x^2 + 8x^4$$

$$\boxed{3x^4 - 5x^2 + 6x}$$

$$10) (2x) + 5x^3 + 7x^4 + 8 + (7x - 5) + 8x^3 - (8x^2 - 7x^4 + 5x)$$

$$14x^4 + 13x^3 - 8x^2 + 4x - 13$$

$$11) (4x + 3)(8x^2 - 6x - 6)$$

	$8x^2$	$-6x$	$-6$
$4x$	$32x^3$	$-24x^2$	$-24x$
$3$	$24x^2$	$-18x$	$-18$

$$\boxed{32x^3 - 42x - 18}$$

$$12) (8x^2 + 2x - 2)(7x^2 - 6x + 7)$$

	$8x^2$	$2x$	$-2$
$7x^2$	$56x^4$	$14x^3$	$-14x^2$
$-6x$	$-48x^3$	$-12x^2$	$12x$
$7$	$56x^2$	$14x$	$-14$

$$\boxed{56x^4 - 34x^3 + 30x^2 + 26x - 14}$$

Describe the end behavior of each polynomial function.

$$13) 4x^6 - 2x^3 + 5x^2 - 1$$

$$\text{as } x \rightarrow -\infty, f(x) \rightarrow \infty \quad \text{as } x \rightarrow \infty, f(x) \rightarrow \infty$$

$$14) -x^3 - 9$$

$$\text{as } x \rightarrow -\infty, f(x) \rightarrow \infty \quad \text{as } x \rightarrow \infty, f(x) \rightarrow -\infty$$

$$15) -3x^4 - 5x^3 + 8x$$

$$\text{as } x \rightarrow -\infty, f(x) \rightarrow -\infty \quad \text{as } x \rightarrow \infty, f(x) \rightarrow -\infty$$

$$16) 9x^{11} - 8x^9 + 7x^3 + 8$$

$$\text{as } x \rightarrow -\infty, f(x) \rightarrow -\infty \quad \text{as } x \rightarrow \infty, f(x) \rightarrow \infty$$

17) A coin is flipped 8 times. What is the probability of getting at least 2 heads?

$$n = 8$$

$$r = 2$$

$$p = 0.5$$

$$q = 0.5$$

$$1 - [{}^8C_0 (0.5)^0 (0.5)^8 + {}^8C_1 (0.5)^1 (0.5)^7]$$

$$P(\geq 2) = 1 - (P(0) + P(1))$$

$$\boxed{0.96484375}$$

$$n = 15 \quad r = 12 \quad p = 0.2 \quad q = 0.8$$

18) You just took your Language Arts Benchmark. Each question had 5 answer choices. What is the probability you got exactly 12 of the 15 questions correct?

$${}_{15}C_{12} (.2)^{12} (.8)^3 = \boxed{.000000954}$$

Identify the following transformations:

<p>19) <math>f(x) = \sqrt[3]{-4x-16} + 8</math>  <math>\sqrt{-4(x+4)} + 8</math></p> <ul style="list-style-type: none"> <li>• reflect over y axis</li> <li>• horizontal shrink by 1/4</li> <li>• left 4</li> <li>• up 8</li> </ul>	<p>20) <math>f(x) = \left \frac{1}{6}x\right  - 4</math></p> <ul style="list-style-type: none"> <li>• horizontal stretch by 6</li> <li>• down 4</li> </ul>	<p>21) <math>f(x) = \frac{1}{8}\sqrt{-x+9}</math>  <math>f(x) = \frac{1}{8}\sqrt{-1(x-9)}</math></p> <ul style="list-style-type: none"> <li>• vertical shrink by 1/8</li> <li>• reflect over y</li> <li>• right 9</li> </ul>	<p>22) <math>f(x) = -8\sqrt{x+9}</math></p> <ul style="list-style-type: none"> <li>• reflect over x-axis</li> <li>• left 9</li> <li>• vertical stretch by 8</li> </ul>
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Write the domain and range of the following functions:

23)  $f(x) = \sqrt[3]{x-6} + 8$

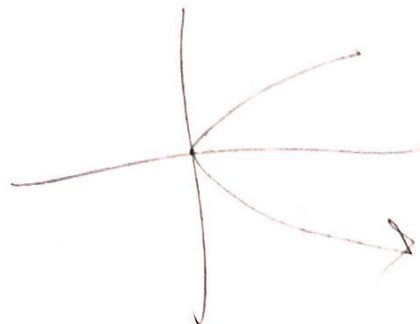
D:  $\mathbb{R}$   
R:  $\mathbb{R}$

24)  $f(x) = \sqrt{3(x-2)} + 8$

$(2, 8)$   
Domain  $x \geq 2$   
Range  $y \geq 8$

25)  $f(x) = -\sqrt{x+3} - 4$

$(-3, -4)$   
Domain  $x \geq -3$   
 $y \leq -4$



SBM #3 Review B

Classify each conic. Then, rewrite into standard form equation of each conic.

1)  $x^2 + y^2 + 26x + 12y + 196 = 0$

2)  $4x^2 + y^2 + 20y - 96 = 0$

$4x^2 + y^2 + 20y + 100 = 96 + 100$

$x^2 + 26x + 169 + y^2 + 12y + 36 = -196 + 169 + 36$   
 $(x+13)^2 + (y+6)^2 = 9$  **Circle**  
 C(-13, -6)  
 r=3

$\frac{4x^2}{196} + \frac{(y+10)^2}{196} = \frac{196}{196}$   
 $\frac{x^2}{49} + \frac{(y+10)^2}{196} = 1$  **ellipse**

3)  $4x^2 - y^2 - 8x + 18y - 177 = 0$

4)  $x^2 + 2x + y - 6 = 0$

$4(x^2 - 2x + 1) - (y^2 - 18y + 81) = 177 + 4 - 81$   
 $4(x-1)^2 - (y-9)^2 = 100$   
 $\frac{(x-1)^2}{25} - \frac{(y-9)^2}{100} = 1$  **hyperbola**

$x^2 + 2x = -y + 6$   
 $x^2 + 2x + 1 = -y + 6 + 1$   
 $(x+1)^2 = -y + 7$   
 $-y + 7 = (x+1)^2$   
 $-y = (x+1)^2 - 7$   
 $y = -(x+1)^2 + 7$  **parabola**

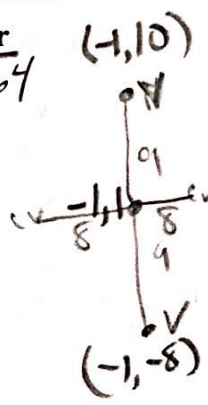
Identify the center, vertices, co-vertices, foci, length of the major axis, and length of the minor axis of each.

5)  $\frac{(x+2)^2}{100} + \frac{(y+2)^2}{49} = 1$

center (-2, -2)  
 a=10 b=7  
 C =  $\sqrt{100-49}$   
 CV (-2, 9) (-2, -5)  
 Foci (-2- $\sqrt{51}$ , -2) (-2+ $\sqrt{51}$ , -2)  
 length of major axis: 20  
 length of minor axis: 14

6)  $\frac{(x+1)^2}{64} + \frac{(y-1)^2}{81} = 1$

C(-1, 1) C =  $\sqrt{81-64}$   
 CV (-1, 10) (-1, -8)  
 Foci (-1, 1+ $\sqrt{17}$ ) (-1, 1- $\sqrt{17}$ )  
 length of major axis: 18  
 length of minor axis: 16



Identify the vertices and foci of each.

7)  $\frac{(x-6)^2}{49} - \frac{(y+9)^2}{100} = 1$

a=7 b=10  
 V(-1, -9) (13, -9)  
 F(6- $\sqrt{149}$ , -9) (6+ $\sqrt{149}$ , -9)

8)  $\frac{(y-4)^2}{25} - \frac{(x-6)^2}{81} = 1$

V(6, 9) (6, -1)  
 Foci (6, 4+ $\sqrt{106}$ ) (6, 4- $\sqrt{106}$ )

Identify the vertex, focus, axis of symmetry, and directrix of each.

9)  $x = -(y-7)^2 - 5$   
 $V(-5, 7)$   
 $F(-5, 7)$   
 Directrix:  $x = -\frac{19}{4}$

$\frac{1}{4p} = -\frac{1}{1}$   
 $1 = -4p$   
 $-\frac{1}{4} = p$

10)  $y = \frac{5}{8}(x+6)^2 + 1$   
 $V(-6, 1)$   
 AOS:  $x = -6$   
 Directrix:  $y = \frac{3}{5}$

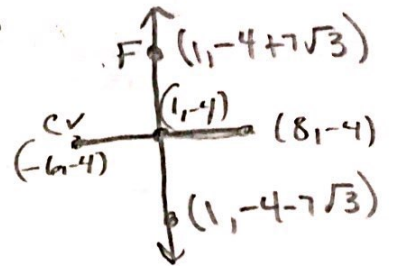
$V(-6, 1)$   
 $\frac{1}{4p} = \frac{5}{8}$   
 $8 = 20p$   
 $\frac{8}{20} = p$   
 $\frac{2}{5} = p$

Use the information provided to write the standard form equation of each ellipse.

11) Foci:  $(1, -4 + 7\sqrt{3}), (1, -4 - 7\sqrt{3})$   
 Co-vertices:  $(8, -4), (-6, -4)$

$C(1, -4)$   $b = 7$   $c = 7\sqrt{3}$   
 $(7\sqrt{3})^2 = a^2 - 49$   
 $147 = a^2 - 49$   
 $196 = a^2$

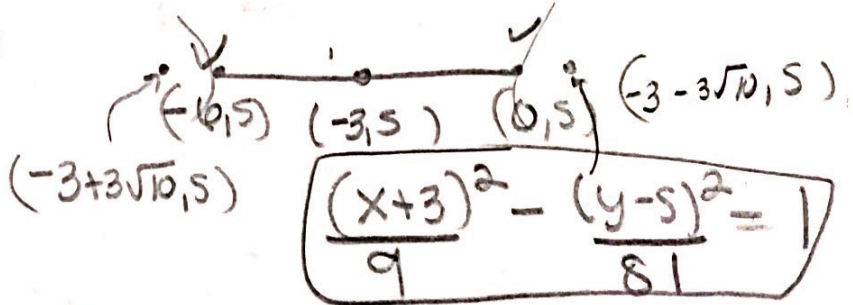
$\frac{(x-1)^2}{49} + \frac{(y+4)^2}{196} = 1$



Use the information provided to write the standard form equation of each hyperbola.

12) Vertices:  $(0, 5), (-6, 5)$   
 Foci:  $(-3 + 3\sqrt{10}, 5), (-3 - 3\sqrt{10}, 5)$

$a = 3$   $C(-3, 5)$   
 $c = 3\sqrt{10}$   
 $(3\sqrt{10})^2 = 9 + b^2$   
 $9 \cdot 10 = 9 + b^2$   
 $90 = 9 + b^2$   
 $81 = b^2$



$\frac{(x+3)^2}{9} - \frac{(y-5)^2}{81} = 1$

Evaluate each determinant.

13)  $\begin{vmatrix} -1 & 2 \\ -4 & -1 \end{vmatrix}$   
 $1 - (-8)$   
 $9$

14)  $\begin{vmatrix} 0 & -4 & 3 & 0 & -4 \\ -3 & -1 & 5 & -3 & -1 \\ -4 & -5 & 1 & -4 & -5 \end{vmatrix}$   
 $(0 + 80 + 45) - (12 + 0 + 12)$   
 $125 - 24$   
 $101$

$$\begin{pmatrix} 1 \\ 4 \end{pmatrix} \begin{pmatrix} -30 \\ 17 \end{pmatrix}$$

Find the inverse of each matrix.

15)  $\begin{bmatrix} 5 & -2 \\ 9 & 9 \end{bmatrix}$   $\det = 45 - (-18) = 63$

$$\frac{1}{63} \begin{bmatrix} 9 & 2 \\ -9 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 9/63 & 2/63 \\ -9/63 & 5/63 \end{bmatrix}$$

$$\begin{bmatrix} 1/7 & 2/63 \\ -1/7 & 5/63 \end{bmatrix}$$

Solve each equation or state if there is no unique solution.

16)  $\begin{bmatrix} 10 & 2 \\ -7 & -1 \end{bmatrix} A = \begin{bmatrix} -30 & 4 \\ 17 & -2 \end{bmatrix}$

$\det = -10 - (-14) = -10 + 14 = 4$

$A^{-1} = \square$

inverse  $\frac{1}{4} \begin{bmatrix} -1 & -2 \\ 7 & 10 \end{bmatrix}$

$$\begin{bmatrix} -1/4 & -1/2 \\ 7/4 & 5/2 \end{bmatrix}$$

$$\begin{bmatrix} -1/4 & -1/2 \\ 7/4 & 5/2 \end{bmatrix} \begin{bmatrix} -30 & 4 \\ 17 & -2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1+1 \\ -10 & 7+-5 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 0 \\ -10 & 2 \end{bmatrix}$$

Simplify. Write "undefined" for expressions that are undefined.

17)  $\begin{bmatrix} -6 \\ 0 \\ 6 \\ 4 \end{bmatrix} + \begin{bmatrix} -4 & 3 \\ 5 & 5 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 5 \\ -6 \\ -6 \\ 0 \end{bmatrix}$

undefined

18)  $2 \begin{bmatrix} 2 & 1 \\ -5 & -6 \\ 2 & 6 \end{bmatrix}$

$$\begin{bmatrix} 4 & 2 \\ -10 & -12 \\ 4 & 12 \end{bmatrix}$$

19)  $\begin{bmatrix} -3 & -5 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} -6 & 4 & -4 \\ 6 & 1 & -4 \end{bmatrix} \begin{bmatrix} -12 & -17 & 32 \\ -24 & 11 & -8 \end{bmatrix}$

$$\begin{bmatrix} 18-30 & -12-5 & 12+20 \\ -18-6 & 12-1 & -12+4 \end{bmatrix}$$

20)  $\begin{bmatrix} 3 & 2 & -3 \\ -2 & 1 & -4 \end{bmatrix} \cdot \left( \begin{bmatrix} -5 & 2 \\ -2 & -4 \\ -3 & 5 \end{bmatrix} - \begin{bmatrix} -4 & -6 \\ 6 & -5 \\ -4 & -1 \end{bmatrix} \right)$

$$\begin{bmatrix} 3 & 2 & -3 \\ -2 & 1 & -4 \end{bmatrix} \cdot \begin{bmatrix} -1 & 8 \\ -8 & 1 \\ 1 & 6 \end{bmatrix}$$

$$\begin{bmatrix} -3-16-3 & 24+2-18 \\ 2-8-4 & -16+1-24 \end{bmatrix} = \begin{bmatrix} -22 & 8 \\ -10 & -39 \end{bmatrix}$$



SBM 3 Review C

Identify the domain and range of each. List the transformations from the parent function.

1)  $y = -3\sqrt{x-4}$   
 (4,0) →  
 • reflect over x-axis  
 • vertical stretch by 3  
 • right 4

Domain:  $x \geq 4$   
 Range:  $y \geq 0$

2)  $y = \sqrt[3]{4x+4} + 2$   
 $\sqrt[3]{4(x+1)} + 2$   
 • horizontal shrink by  $\frac{1}{4}$   
 • left 1  
 • up 2

Domain:  $\mathbb{R}$   
 Range:  $\mathbb{R}$

3)  $y = \sqrt{-x-3}$   
 $y = \sqrt{-1(x+3)}$   
 • reflect over y-axis  
 • left 3

Domain  $x \leq -3$   
 Range  $y \geq 0$

4)  $y = \frac{2}{5}\sqrt[3]{x} - 9$   
 • vertical shrink by  $\frac{2}{5}$   
 • down 9

Domain:  $\mathbb{R}$   
 Range:  $\mathbb{R}$

Solve each equation. Remember to check for extraneous solutions.

5)  $(\sqrt[3]{2x+14})^3 = (4)^3$   
 $2x+14 = 64$   
 $2x = 50$   
 $x = 25$

6)  $5\sqrt[3]{-6-n} = 2\sqrt[3]{3n+26}$   
 $125(-6-n) = 8(3n+26)$   
 $-750 - 125n = 24n + 208$   
 $-750 = 149n + 208$   
 $-958 = 149n$   
 $\frac{-958}{149} = n$   
 $n = -6.43$

$5\sqrt[3]{-6-6.43} = 2\sqrt[3]{3(6.4)+26}$   
 $5\sqrt[3]{-12.43} = 2\sqrt[3]{24.8}$   
 $3.77 = 3.7$

7)  $(\sqrt{72-m})^2 = (m)^2$   
 $\sqrt{72+9} = -9$   
 $72-m = m^2$   
 $m^2 + m - 72 = 0$   
 $(m+9)(m-8) = 0$   
 $m = 8$

8)  $b+4 = \sqrt{6b+15}$   
 $(b+4)^2 = (\sqrt{6b+15})^2$   
 $b^2 + 8b + 16 = 6b + 15$   
 $b^2 + 2b + 1 = 0$   
 $(b+1)(b+1) = 0$   
 $b = -1$



Evaluate each at  $f(-5)$ ,  $f(-1)$ ,  $f(0)$ , and  $f(1)$

$$9) f(x) = \begin{cases} (x+2)^4, & x \leq -1 \\ 4-x^3, & x > -1 \end{cases} \quad f(-1) = (-1+2)^4 = (1)^4$$

$$f(-5) = (-5+2)^4 = (-3)^4 = 81$$

$$f(-1) = 1$$

$$f(0) = 4 - 0^3 = 4$$

$$f(1) = 4 - 1^3 = 3$$

$$10) h(x) = \begin{cases} \sqrt{-x}, & x < -4 \\ x-1, & -4 \leq x \leq 0 \\ 4-x^2, & x > 0 \end{cases}$$

$$h(-5) = \sqrt{-(-5)} = \sqrt{5}$$

$$h(-1) = -1 - 1 = -2$$

$$h(0) = 0 - 1 = -1$$

$$h(1) = 4 - 1^2 = 3$$

Find the inverse of each function.

$$11) f(x) = -2x - 2$$

$$x = -2y - 2$$

$$\frac{x+2}{-2} = \frac{-2y}{-2}$$

$$f^{-1}(x) = \frac{x}{-2} - 1$$

$$12) g(x) = \sqrt[3]{x-2} + 1$$

$$x = \sqrt[3]{y-2} + 1$$

$$x-1 = \sqrt[3]{y-2}$$

$$(x-1)^3 = y-2$$

$$(x-1)^3 + 2 = y$$

$$g^{-1}(x) = (x-1)^3 + 2$$

Perform the indicated operation.

$$13) f(t) = 4t + 3$$

$$g(t) = -t^2 + 2$$

Find  $(f \cdot 2g)(t)$

$$f(t) \cdot 2g(t)$$

$$(4t+3) \cdot 2(-t^2+2)$$

$$(4t+3)(-2t^2+4)$$

$$-8t^3 + 16t - 6t^2 + 12$$

$$-8t^3 - 6t^2 + 16t + 12$$

$$14) g(x) = x^2 - 5$$

$$f(x) = 3x + 5$$

Find  $(g - 5f)(x)$

$$g(x) - 5f(x)$$

$$x^2 - 5 - 5(3x+5)$$

$$x^2 - 5 - (15x + 25)$$

$$x^2 - 5 - 15x - 25$$

$$x^2 - 15x - 30$$

15)  $g(t) = 2t^2 + t - 3$   
 $f(t) = 4t^2 - 5t + 1$   
 Find  $\left(\frac{g}{f}\right)(t)$

$$\frac{2t^2 + t - 3}{4t^2 - 5t + 1}$$

$$\begin{array}{r} 4t^2 \\ -4 \\ \hline -1 \\ -5t \end{array}$$

$$\begin{array}{l} 4t^2 - 4t - t + 1 \\ 4t(t-1) - 1(t-1) \\ (4t-1)(t-1) \end{array}$$

17)  $f(t) = 2t$   
 $g(t) = -4t - 4$   
 Find  $(f \circ g)(-9)$

$$g(-9) = -4(-9) - 4$$

$$= 36 - 4$$

$$g(-9) = 32$$

$$f(32) = 2(32)$$

$$= \boxed{64}$$

19)  $h(n) = 2n^2 + 2$   
 $g(n) = n - 1$   
 Find  $h(g(n))$

$$\begin{aligned} h(n-1) &= 2(n-1)^2 + 2 \\ &= 2(n^2 - 2n + 1) + 2 \\ &= 2n^2 - 4n + 2 + 2 \end{aligned}$$

$$\boxed{h(n-1) = 2n^2 - 4n + 4}$$

$$\begin{array}{r} -6t \\ +3 \\ \hline -2 \\ t \end{array}$$

$$\begin{aligned} 2t^2 + 3t - 2t - 3 \\ t(2t+3) - 1(2t+3) \\ (t-1)(2t+3) \\ \hline (t-1)(4t-1) \\ \hline \boxed{\frac{2t+3}{4t-1}} \end{aligned}$$

16)  $f(n) = 4n - 2$   
 $g(n) = n + 3$   
 Find  $f(g(n))$

$$\begin{aligned} f(n+3) &= 4(n+3) - 2 \\ &= 4n + 12 - 2 \\ \boxed{f(n+3) = 4n + 10} \end{aligned}$$

18)  $f(a) = a + 1$   
 $g(a) = 2a$   
 Find  $f(g(10))$

$$g(10) = 2(10)$$

$$= 20$$

$$f(20) = 20 + 1$$

$$= \boxed{21}$$

20)  $f(x) = 2x - 1$   
 $g(x) = x^3 - 3x$   
 Find  $f(g(x))$

$$\begin{aligned} f(x^3 - 3x) &= 2(x^3 - 3x) - 1 \\ &= 2x^3 - 6x - 1 \\ \boxed{2x^3 - 6x - 1} \end{aligned}$$